COMPOSITE PHASE AND PHASE-BASED GABOR ELEMENT AGGREGATION

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ABSTRACT

This paper describes how the phase, obtained by Gabor filtering an image, can be used to aggregate related Gabor elements (simple features identified by peaks in the Gabor magnitude). This phase-based feature grouping simplifies the perennial problem of target/background segmentation because we need to only determine if the aggregate feature is target or background, rather than determine the status of each feature independently. Since the phase from a single quadrature Gabor output cannot tolerate large changes in orientation, a new local measure, referred to as the *composite phase*, has been developed. It is a combination of the filter responses from multiple orientations which allows the phase to follow contours with large changes in orientation. A constant composite phase contour is used to connect related Gabor elements that would otherwise appear separated within the magnitude response.

1. INTRODUCTION

Bandpass filtering an image using a set of quadrature Gabor kernels highlights interesting image features [1]. These features, referred to as "Gabor elements," are identified as peaks in the Gabor magnitude response. An individual Gabor element can be useful for selecting a region-of-interest that potentially contains a target. However, an isolated Gabor element does not provide adequate information to recognize a viewed target because the Gabor element is too localized in terms of position and orientation. Grouping Gabor elements that belong to a common target or structure can provide the necessary diversity in position and orientation to distinguish a target from background clutter. Contour extraction can be useful for grouping related Gabor elements found in low resolution infrared images.

A new local measure, the "composite phase," is introduced for contour extraction. The composite phase uses information from all basis orientations (usually four), but one common frequency. It can be used to extract contours (or portions of contours) that are described as "single-valued polar curves" (see section 2.1).

2. GABOR-BASED LOCAL MEASURES

In this paper, the basis filter set is comprised of quadrature Gabor kernels [2], which are given by

$$G_+(x,y,\omega_i,\phi_i) = g(x,y)\cos[\omega_i(x\cos\phi_i + y\sin\phi_i)],$$
$$G_-(x,y,\omega_i,\phi_i) = g(x,y)\sin[\omega_i(x\cos\phi_i + y\sin\phi_i)],$$
$$g(x,y) = \exp\left(-\frac{\lambda^2\omega_i^2(x^2+y^2)}{4\pi}\right), \quad (1)$$

where x and y are the horizontal and vertical image coordinates, respectively; ω_i and ϕ_i are the modulation frequency and orientation, respectively; and λ is a constant. The spatial and spectral characteristics for a set of four polar-spaced Gabor functions are shown in figures 1 (a) and 1 (b), respectively.

The spatially sampled output of the Gabor filter is referred to as a *Gabor coefficient*, and is given by

$$a_\pm(i) = \iint I(x,y)G_\pm(x_i - x, y_i - y, \omega_i, \phi_i)dx\,dy, \quad (2)$$

where $I(x,y)$ is the input image. The local magnitude m and the phase θ, obtained from the quadrature Gabor coefficients, are given by

$$m(x_i,y_i,\omega_i,\phi_i) = \sqrt{a_+^2(i) + a_-^2(i)}, \quad (3)$$
$$\theta(x_i,y_i,\omega_i,\phi_i) = \arctan\left[\frac{a_-(i)}{a_+(i)}\right]. \quad (4)$$

Other local measures [1] are obtained by combining the magnitude responses from basis channels with different orientations, but a common frequency. The *marginal magnitude*, m_c, is given by

$$m_c(x_i,y_i,\omega_i) = \sum_{\phi_i} m(x_i,y_i,\omega_i,\phi_i). \quad (5)$$

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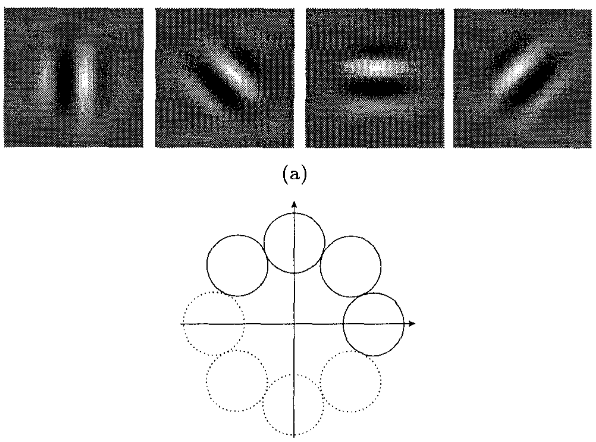


(a)

(b)

Figure 1: Gabor functions for four orientations. (a) Spatial domain. Only sine component shown. Orientations from right to left are ϕ_0, ϕ_1, ϕ_2, ϕ_3. (b) Spectral domain. Orientations, counter-clockwise from horizontal, are ϕ_0, ϕ_1, ϕ_2, ϕ_3, $-\phi_0$, $-\phi_1$, $-\phi_2$, $-\phi_3$. Mirrored channels are shown as dotted lines.

The *dominant spectral orientation*, ϕ_d, is given by

$$\phi_d(x_i, y_i, \omega_i) = \frac{1}{2} \arctan \left\{ \frac{\sum_{\phi_i} m(\phi_i) \sin[2\phi_i]}{\sum_{\phi_i} m(\phi_i) \cos[2\phi_i]} \right\}. \quad (6)$$

The variance in the orientation, or *normalized minimum moment of inertia*, is given by

$$J_{nmoi}(x_i, y_i, \omega_i) = \frac{\sum_{\phi_i} m(\phi_i) \sin^2[\phi_i - \phi_d(x_i, y_i, \omega_i)]}{\sum_{\phi_i} m(x_i, y_i, \omega_i, \phi_i)}. \quad (7)$$

2.1. Composite Phase

The phase response, $\theta(x_i, y_i, \omega_i, \phi_i)$, is useful for target detection because lines and edges often appear as equiphase contours [3]. However, phase contours obtained from a single Gabor channel will only follow the segments of the image curve whose orientation falls within the channel passband. The *composite phase* combines responses from each orientation enabling it to follow long phase contours that have large changes in orientation.

The composite phase, in this work, is given by

$$\theta_c(x_i, y_i, \omega_i) = \arctan \left[\frac{\sum_{\phi_i} \cos(\phi_i - \phi_r) a_-(i)}{\sum_{\phi_i} |\cos(\phi_i - \phi_r)| a_+(i)} \right], \quad (8)$$

where

$$\phi_r(x_i, y_i) = \arctan \left[\frac{y_i - y_r}{x_i - x_r} \right], \quad (9)$$

and (x_i, y_i) and (x_r, y_r) are the coordinates of a given contour point and the predefined reference point, respectively. There are two note-worthy properties of θ_c: the "sliding" orientation reference $\phi_r(x_i, y_i)$ and the "shading" of $\cos(\phi_i - \phi_r)$.

Equation (8) requires a "sliding" orientation reference $\phi_r(x_i, y_i)$ to avoid a phase inversion problem. Consider the case of an image contour whose orientation changes significantly, such as the boundary enclosing a bright region. If a fixed ϕ_r is used throughout, phase inversions would appear as one moves along the contour due to an artifact of the spectral representation of the orientation. In the spectral domain, each Gabor channel is mirrored through the origin, so that the orientations ϕ_i and $\phi_i + \pi = -\phi_i$ are represented by the same channel. If one were to travel in $\frac{\pi}{4}$ radian increments from orientation ϕ_0, one would visit the channels in the following order: ϕ_0, ϕ_1, ϕ_2, ϕ_3, $-\phi_0$, $-\phi_1$, $-\phi_2$, and $-\phi_3$. Using the above-mentioned orientation assignment, the composite phase would undergo a phase inversion as the dominant channel changed from ϕ_0 to $-\phi_3$ or from ϕ_3 to $-\phi_0$. The phase inversion is avoided by changing the sign of the coefficient $a_-(i)$ when the basis orientation, ϕ_i, differs from $\phi_r(x_i, y_i)$ by more than $|\frac{\pi}{2}|$ radians. The sign of the $\cos(\phi_i - \phi_r)$ term in (8) adjusts the orientation assignment accordingly, steering the inversion away from $\phi_r(x_i, y_i)$. Note that the inversion does not affect a_+, which means that a contour of $\theta_c = 0$ or π (a line as opposed to an edge contour) is not affected.

The "shading term" of $|\cos(\phi_i - \phi_r)|$ is incorporated into (8) to make the composite phase estimate more robust. Without the shading term, the composite phase estimate could be adversely affected by responses that are orthogonal to $\phi_r(x_i, y_i)$. For example, if the reference orientation at a given point on the contour is $\phi_r = 0$, a large response from channel $\phi_2 = \frac{\pi}{2}$ would cause the composite phase to be unstable. That is, a small change in ϕ_r would cause a sign change in the term $(\phi_2 - \phi_r)$. Shading suppresses such instabilities.

The orientation reference ϕ_r, defined by (9), is the orientation of a ray emanating from the reference point (x_r, y_r) towards a contour point (x_i, y_i). For a given (x_r, y_r), the resulting equi-phase contours will follow image curves (or portions of) that can be described by single-valued polar equations:

$$\rho_r(x_i, y_i) = [(x_i - x_r)^2 + (y_i - y_r)^2]^{0.5} = f(\phi_r). \quad (10)$$

Convex and star-shaped curves can be represented as single-valued polar curves, if the reference point (x_r, y_r)

is properly chosen. For many image curves, including convex curves, there is a large set of reference points that satisfy the single-valued polar equation requirement; however, for the composite phase defined by (8), the best reference point is selected such that the reference orientation ϕ_r is close to the local dominant orientation, ϕ_d, throughout the length of the contour.

To obtain a reference point that makes $\phi_r \approx \phi_d$ throughout the contour, the sum of *excess moment of inertia* is minimized. The excess moment of inertia at a contour point (x_i, y_i), about its reference orientation ϕ_r, is given by

$$\begin{aligned} e_{moi}(\omega_i, \phi_r) &= [\sum_{\phi_i} m(x_i, y_i, \omega_i, \phi_i) \sin^2(\phi_i - \phi_r)] \\ &- m_c(x_i, y_i, \omega_i) \cdot J_{nmoi}(x_i, y_i, \omega_i). \end{aligned}$$

If the dominant orientation (ϕ_d) has already been calculated, an alternative form for e_{moi} is given by

$$e_{moi} = m_c(1 - 2 \cdot J_{nmoi}) \sin^2(\phi_d - \phi_r). \tag{11}$$

The cost function to be minimized is given by

$$J_r(\omega_i) = \sum_{\phi_r(x_i, y_i)} \rho_r^2 e_{moi}(\omega_i, \phi_r). \tag{12}$$

Since the "optimal" reference point $(x_r,\ y_r)$ is obtained using an iterative search, the gradient is necessary. The first partial derivatives are given by

$$\frac{\partial J_r}{\partial x_r} = \sum -2(x_i - x_r) \cdot e_{moi} + \sum \frac{\partial e_{moi}}{\partial \phi_r}[y_i - y_r], \tag{13}$$

$$\frac{\partial J_r}{\partial y_r} = \sum -2(y_i - y_r) \cdot e_{moi} + \sum \frac{\partial e_{moi}}{\partial \phi_r}[-x_i + x_r], \tag{14}$$

where

$$\frac{\partial e_{moi}}{\partial \phi_r} = m_c(2 \cdot J_{nmoi} - 1) \sin[2(\phi_d - \phi_r)]. \tag{15}$$

The update equation for the reference point coordinates is given by

$$\begin{bmatrix} x_r \\ y_r \end{bmatrix}_{i+1} = \begin{bmatrix} x_r \\ y_r \end{bmatrix}_i - \beta \cdot [\frac{\partial J_r}{\partial x_r} \ \frac{\partial J_r}{\partial y_r}]^T, \tag{16}$$

where β is a positive constant, referred to as the "convergence factor." For the cost function (12), a suitable convergence factor is

$$\beta = \frac{1}{\sum_{\phi_r(x_i, y_i)} m_c(1 - 2 \cdot J_{nmoi})}. \tag{17}$$

3. GROUPING GABOR ELEMENTS

The composite phase is useful for grouping Gabor elements. In this work, Gabor elements are grouped together if they exhibit phase coherence in the form of a constant composite phase. To achieve this grouping, it is necessary to extract constant phase contours that contain many Gabor elements. Since Gabor elements coincide with peaks in the magnitude response, long contours passing through high magnitude regions are considered significant.

Significant phase contours are extracted as follows.

1. Pick a reference Gabor element. The contour of interest is the one that passes through this reference Gabor element.
2. Select an initial reference point, (x_r, y_r), from which the composite phase is calculated.
3. Select the reference phase $\theta_{c,ref}$. To extract the contour, trace along the constant phase from the reference Gabor element.
4. Re-calculate the reference point, (x_r, y_r), using the extracted contour. Re-calculate the composite phase if (x_r, y_r) has shifted significantly.

The selection of the reference Gabor element is not critical because, in most applications, all Gabor elements will either be selected as a reference element or grouped with a previously selected reference element. The selection of the initial reference point is obtained using the center of gravity of the magnitude. This requires coarse region-of-interest mask be placed around the reference Gabor element. The reference phase, $\theta_{c,ref}$, is set equal to the phase of the reference Gabor element; however, the choice of $\theta_{c,ref}$ is not critical for the purpose of grouping Gabor elements. Note that the phase contour, and hence the grouping of Gabor elements, can change during the re-calculation of (x_r, y_r) because the new reference point may cause an image curve to be split into many single-valued polar curves, or may alter the location of such splits.

4. RESULTS AND CONCLUSION

This section demonstrates the use of the composite phase to group Gabor elements that belong to a common contour. The test image is a forward looking infrared (FLIR) image of a tank, shown in Figure 2 (a). The objective of this test is to extract and group Gabor elements that belong to the image contours corresponding to the tank turret (boundary of the bright blob in Figure 2 (a)). Figure 3 shows the magnitude responses

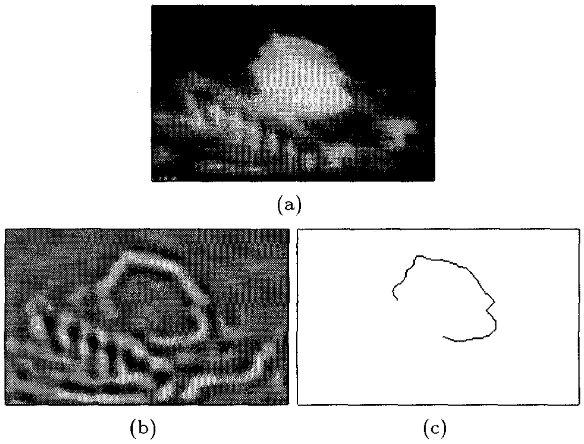
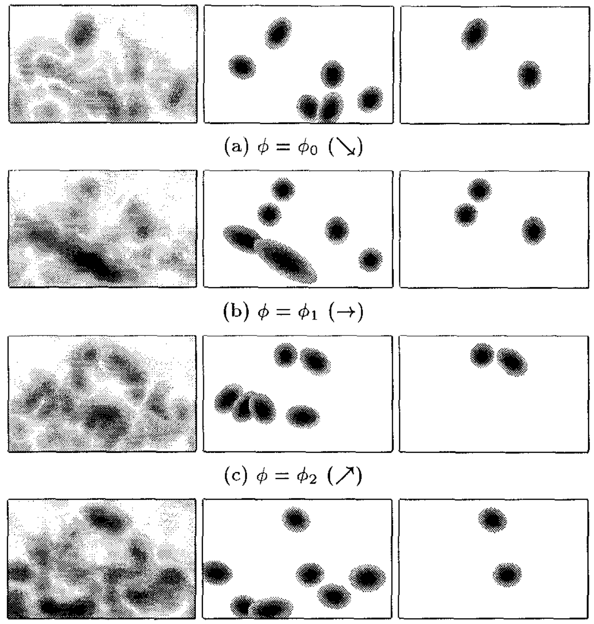


Figure 2: Contour Extraction. (a) FLIR image of a tank. (b) Sine of composite response, when reference point, (x_r, y_r), is at the center of the image. (c) Contour for phase $\theta_{c,ref} = -\frac{\pi}{2}$.

(left-most column) and detected features (center column) for the four basis orientations. Notice that there are many detected Gabor elements that are not part of the tank turret and some Gabor elements that are not part of the tank.

Grouping Gabor elements using constant phase contours can reject clutter. Figure 2 demonstrates how a contour is extracted. In this example, the image size is (228,141), the origin (0,0) is at the upper-left corner, the selected reference Gabor element is found near the top of figure 3 (d) at image coordinates (112,44), the reference point is $(x_r, y_r) = (114, 70)$ (that is, the center of the image), and the reference phase is $-\frac{\pi}{2}$. The sine response, used to calculate the composite phase, is shown in Figure 2 (b). The contour containing the reference Gabor element is shown in Figure 2 (c). The updated reference point for the extracted contour is (128,68). Since the shift in the reference point is small, the composite phase does not have to be re-calculated. All of the detected Gabor elements belonging to the extracted contour are shown in Figure 3 (right-most column). It is interesting to note that the extracted contour (figure 2 (c)) passed through regions in figure 2 (b) with weak responses, successfully connecting Gabor elements from diverse orientations and spatial positions. This grouping process removes all the non-turret responses, and, hence, simplifies target model indexing as well as target-background segmentation.

In conclusion, a new measure, the composite phase, has been presented that is useful for grouping related



(a) $\phi = \phi_0$ ($\searrow$)

(b) $\phi = \phi_1$ ($\rightarrow$)

(c) $\phi = \phi_2$ ($\nearrow$)

(d) $\phi = \phi_3$ ($\uparrow$)

Figure 3: Magnitude response (*left*), detected features (*center*), and grouped turret features (*right*), of tank for Gabor basis filters with various orientations: (a) $\phi_0 = -\frac{\pi}{4}$, (b) $\phi_1 = 0$, (c) $\phi_2 = \frac{\pi}{4}$, (d) $\phi_3 = \frac{\pi}{2}$ radians. Dark regions indicate a high magnitude or the location/shape of features.

Gabor elements that correspond to a constant phase contour. This grouping aids the detection of target parts and simplifies target-background segmentation.

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5. REFERENCES

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