COMPUTATION OF TWO-DIMENSIONAL COMPLEX CEPSTRUM

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ABSTRACT

A technique based on fitting splines to the phase derivative curve is presented for the efficient and reliable computation of the two-dimensional complex cepstrum. The technique is an adaptive numerical integration scheme and makes use of several computational strategies within the Tribolet's phase unwrapping algorithm. An application of the complex cepstrum in testing the stability of two-dimensional recursive digital filters is considered. Susceptibility of the computation of complex cepstrum to slight changes in the coefficients of a two-dimensional array is studied. Several examples of stable and unstable two-dimensional quarter-plane and non-symmetric half-plane recursive digital filters are presented.

I. INTRODUCTION

The main step in using the technique of homomorphic signal processing is the efficient and reliable computation of the complex cepstrum. The computation of complex cepstrum is of importance because of its use in testing the stability of 2-D recursive (infinite impulse response) digital filters, to solve the blind deconvolution problem in 2-D, to characterise reverberance in 2-D signals etc. (1-3). Dudgeon (4) uses Tribolet's (5) 1-D phase unwrapping algorithm and the recursion equations for the computation of the 2-D complex cepstrum. Bhanu and McClellan (6) have described a technique based on fitting splines to the phase derivative for the efficient and reliable computation of the 1-D complex cepstrum. One common complaint with the use of 1-D phase unwrapping techniques to compute the 2-D complex cepstrum is that if we reverse the order of row and column operations, different results are obtained. The reason for this lies in the incorrect phase unwrapping techniques. In this paper we use an optimized 1-D phase unwrapping technique (6,7) to compute the 2-D complex cepstrum where such errors do not occur. We present several phase unwrapping examples when we use complex cepstrum to check the stability of IIR filters.

II. COMPUTATIONAL DETAILS AND PROPERTIES

Fig. 1 shows a 2-D homomorphic system for convolution. The complex cepstrum is given by,

$$\hat{x}[m,n] = Z^{-1}\,[\log Z(x[m,n])] \qquad (1)$$

It is defined within its region of convergence that includes $|w| = |z| = 1$. Evaluating $\hat{X}(w,z) = \mathrm{Log}[X(w,z)]$ at $w = e^{j\mu}$ and $z = e^{j\nu}$, we get,

$$\hat{X}_R(e^{j\mu},e^{j\nu}) = \log |X(e^{j\mu},e^{j\nu})| \qquad (2)$$
$$\hat{X}_I(e^{j\mu},e^{j\nu}) = \arg\,[X(e^{j\mu},e^{j\nu})]$$

The unwrapped phase can be obtained by integrating the phase derivative. However, because of the inherent truncation error in the numerical integration and the inadequacy of the principal value alone, an adaptive approach is used. The use of bicubic spline interpolation does not seem feasible. In (6,7) we have described the incorporation of various features such as the improvement of the integration rule using splines, efficient computation of DFT at a single frequency off the FFT grid, the need of double precision for certain variables, determination of incremental and consistency thresholds and the estimation of linear phase within the Tribolet's algorithm. The use of these features results in an efficient and reliable phase unwrapping algorithm. There can be several approaches for phase unwrapping (7) when using the 1-D phase unwrapping algorithm to compute the 2-D complex cepstrum. The approach that we have used is to compute the first and second partial deivatives with respect to μ at all the DFT points and with respect to ν along the first column only (7). Computation of these derivatives require the Fourier transforms of $x[m,n]$, $nx[m,n]$, $mx[m,n]$, $n^2x[m,n]$, $m^2x[m,n]$. First we compute the initial phase at the origin (5) and then unwrap the phase along the

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first column using the optimized adaptive phase unwrapping algorithm (6,7). This is followed by the computation of the unwrapped phase for every row. Linear phase components are determined by using the value of the unwrapped phase at π along the axes. These componentes are subtracted from the unwrapped phase to make it periodic. The 2-D FFT at a single frequency off the FFT grid can be efficiently computed by extending the Bonzanigo's modification (6,7) to 2-D.

The following 3 properties of the 2-D complex cepstrum are used in the following sections (7).

1. For a separable sequence the complex cepstrum exists only on the axes.

Before considering the next property, a few definitions follow:

Support of h[m,n] is the set $\{(m,n) \mid h[m,n]\} \neq 0\}$

Non-Symmetric half plane (NSHP) is a region of the form $\{m \geq 0, n \geq 0\} \cup \{m>0, n<0\}$ or their rotations. There are total 8 NSHPs.

Admissible region is a NSHP intersected with a sector.

2. If the support of $\hat{h}[m,n]$ lies in an admissible region, then support of h[m,n] lies in the same admissible region.

3. When the signal x[m,n] is known to be min-min phase (i.e., has no poles or zeros in the region $|w| \geq 1, |z| \geq 1$), the use of recursion equations allow us to compute $\hat{x}[m,n]$ accurately (4,7). However, the use of these equations is inefficient compared to the DFT approach.

III. STABILITY OF IIR DIGITAL FILTERS

A number of attempts (8-10) have been made to formulate algorithm tests for testing the stability of 2-D recursive digital filters. The numerical implementation of these tests is usually inefficient. Moreover, these results are applicable only to the class of 2-D IIR filters which are quadrant causal. The use of complex cepstrum generalises the concept of stability test in the sense that it not only includes quarter-plane filters, but also non-symmetric half-plane filters(2,3) and the implementation is more efficient than other tests when the FFT is employed. The order of the filter can be very high also. Ekstrom and Woods (2) have described a method for checking the stability. They avoid the problem of phase unwrapping by taking the input of a homomorphic system as the autocorrelation of the sequence. This procedure is not computationally as attractive as the method based on computing the complex cepstrum which requires phase unwrapping. Ekstrom and Twogood (3) have presented the stability test which requires phase unwrapping. However, their phase unwrapping is based on the principal value, hence it is not very reliable. Filip (1) got a very poor estimate of the phase while using the phase unwrapping approach based only on the principal value. Our approach for testing the stability is similar to Ekstrom and Twogood (3) but the phase unwrapping technique is different. To check the stability of the filter G(w,z) = N(w,z)/D(w,z), where N(w,z) and D(w,z) are each 2-D polynomials, we compute $\hat{d}[m,n]$ corresponding to the denominator polynomial D(w,z). Now we apply property 2 to check if the support of $\hat{d}[m,n]$ is the same as that of d[m,n]. If yes, then the filter is stable otherwise it is unstable. The DFT implementation gives $\hat{d}_a[m,n]$ which is an aliased version of $\hat{d}[m,n]$. But since the cepstrum decays faster than an exponential, $\hat{d}_a[m,n]$ will be a reasonable approximation of $\hat{d}[m,n]$ for the modest size of 2-D FFTs.

IV. EXAMPLES AND COMMENTS

Example 1 Quarter-plane filter involving separable sequences. This example has been examined by Anderson and Jury (10). The 2-D denominator array d[m,n] is given by,

n ↑		
6	5	1
12	10	2

(→ m)

and the complex cepstrum is $[\log 12]\delta[m,n]$

$$-\frac{1}{m}\left[\left(-\frac{1}{3}\right)^{m}+\left(-\frac{1}{2}\right)^{m}\right]u[m-1]-\frac{\left(-\frac{1}{2}\right)^{n}}{n}u[n-1] \quad (3)$$

The principal value plot is the same as the unwrapped phase shown in fig. 2. The complex cepstrum is shown in fig. 3 and it can be verified by comparing it to the known cepstrum of equation (3). Note that $\hat{d}[m,n]$ exists only on the axes and it has the same support as d[m,n] so the stability is guarnteed. A 64x64 sized FFT is used.

Example 2 Quarter-plane 6th order bandpass filter. This filter has been examined by Ekstrom and Twogood (3). The denominator array of the filter is given by,

n ↑						
0.015626						
0.09375	0.046875					
0.375	0.28125	0.09375				
0.875	0.9375	0.46875	0.109375			
1.5	1.875	1.3125	0.46875	0.09375		
1.5	2.25	1.875	0.09375	0.28125	0.046875	
1.	1.5	1.5	0.875	0.375	0.09375	0.015625

(→ m)

Fig. 4 shows the principal value and fig. 5 the unwrapped phase. Note that the discontinuities because of the modulo 2π operation have been removed. The FFT size used is 64x64. The complex cepstrum is shown in Fig. 6. Its values are verified using the recursion equations. Since the support of the cepstrum lies in the first quadrant, the filter is stable.

Example 3 Non-symmetric half plane filter. Let the filter be given by

$$N(w,z)=1 \text{ and } D(w,z) = 1-\frac{1}{3}z^{-1}-\frac{1}{3}w^{-1}z \quad (4)$$

The complex cepstrum corresponding to equation (4) in closed form is given by,

$$\hat{d}[m,n] = -\binom{2m+n}{m}\frac{(\frac{1}{3})^{2m+n}}{2m+n}, \quad 2m+n \geq 1, \; 0 \leq m \leq 2m+n$$

Fig. 7 shows the principal value and Fig. 8 the unwrapped phase. Note that the jumps introduced by the modulo 2π operation have been removed. The complex cepstrum is shown in Fig. 9. The FFT size used is 64x64. Since the cepstrum and the sequence occupy the same support, the filter is stable.

Example 4 Unstable filter examined by Shanks (8). The denominator array is,

n \ m		
	-0.9	0.5
o	1.	-0.95

Figs. 10 and 11 show the principal value and the unwrapped phase after the removal of the linear phase. Observe that the unwrapped phase is not continuous and the complex cepstrum shown in Fig. 12 does not occupy the same support as the sequence, hence the filter is unstable. Using Huang's test (9) the filter can also be shown to be unstable. FFT size used is 32x32.

Example 5 Unstable filter examined by Shanks (8). The denominator array is,

n \ m			
	0.5	-0.75	0.25
	-1.2	1.8	-0.72
o	1.	-1.5	0.6

Figs. 13-15 show the principal value, unwrapped phase and the complex cepstrum. FFT size used is 64x64. Comments similar to example 4 apply here. In an attempt to observe how the computation of the complex cepstrum is susceptible to the slight changes in the values of the coefficients, we changed the value of d[2,2] in the denominator array of example 4 from 0.25 to 0.29. This case has also been examined by Shanks using contour mapping and has been shown to stable. Figs. 16 and 17 show the unwrapped phase and the complex cepstrum. The unwrapped phase is the same as the phase principal value. Both the cepstrum and the sequence have the first quadrant support, hence the filter is stable. Computed values of the cepstrum are verified by using the recursion equations. FFT size used is 32x32. This example illustrates that the computation of the complex cepstrum is quite sensitive to the stable and unstable cases and the phase unwrapping algorithm is reliable.

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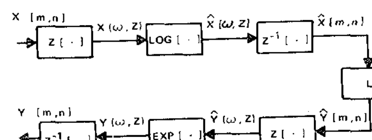

Fig. 1 2-D Homomorphic System

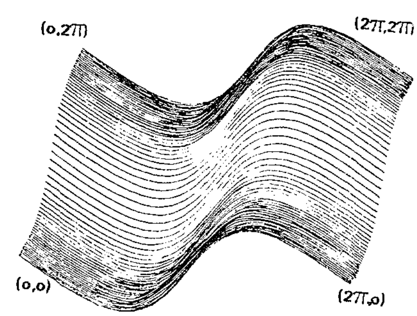

Fig. 2 Unwrapped Phase (Ex. 1)

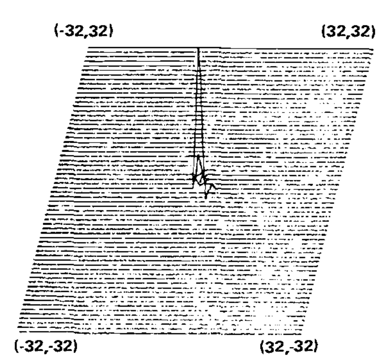

Fig. 3 Complex Cepstrum (Ex. 1)

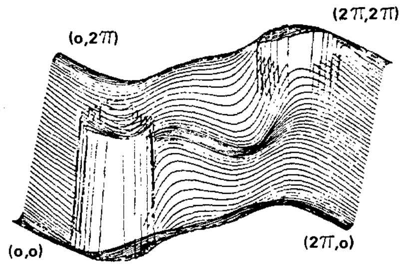

Fig. 4 Principal Value (Ex. 2)

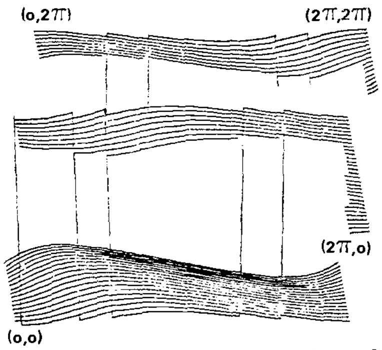

Fig. 7 Principal Value (Ex. 3)

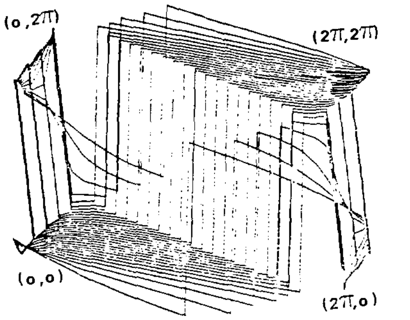

Fig. 10 Principal Value (Ex. 4)

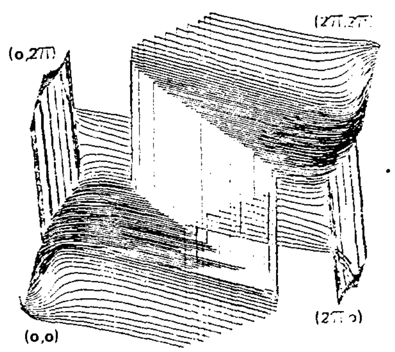

Fig. 13 Principal Value (Ex. 5)

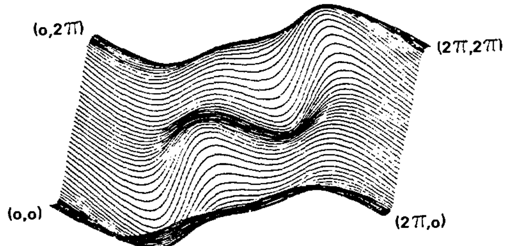

Fig. 5 Unwrapped Phase (Ex. 2)

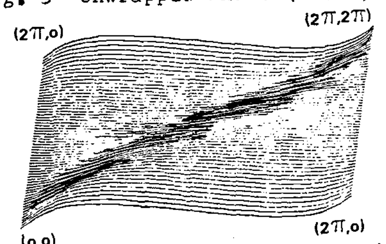

Fig. 8 Unwrapped Phase (Ex. 3)

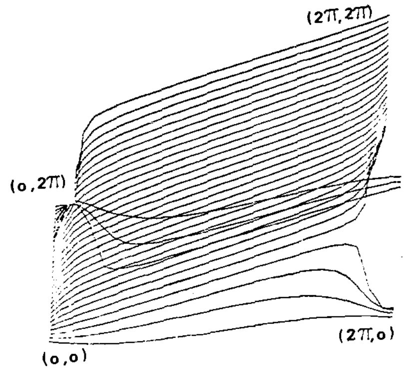

Fig. 11 Unwrapped Phase (Ex. 4)

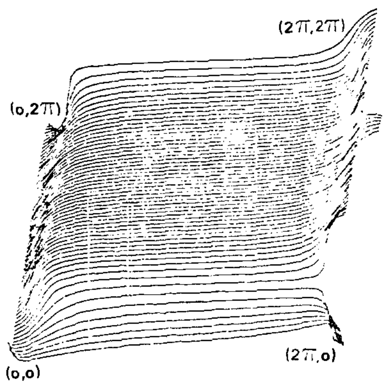

Fig. 14 Unwrapped Phase (Ex. 5)

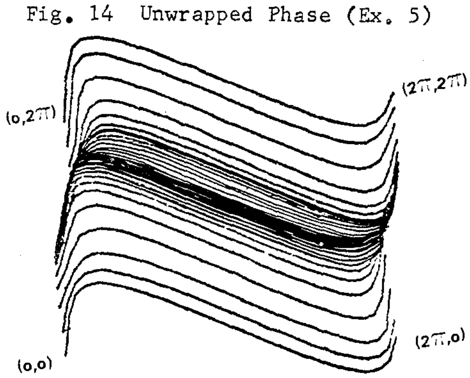

Fig. 16 Unwrapped Phase

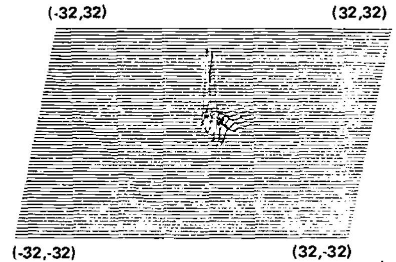

Fig. 6 Complex Ceptrum (Ex. 2)

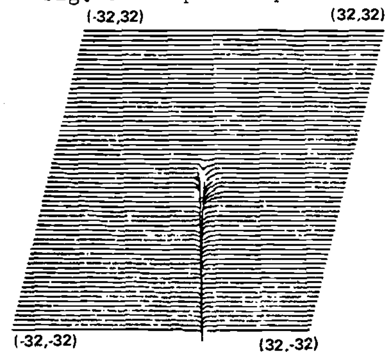

Fig. 9 Complex Cepstrum (Ex. 3)

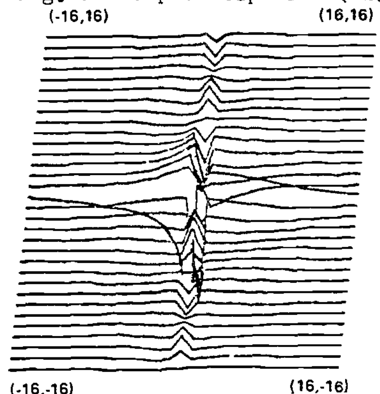

Fig. 12 Complex Cepstrum (Ex. 4)

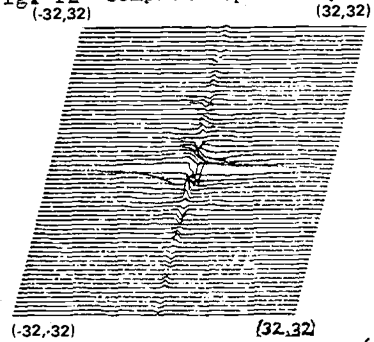

Fig. 15 Complex Cepstrum (Ex. 5)

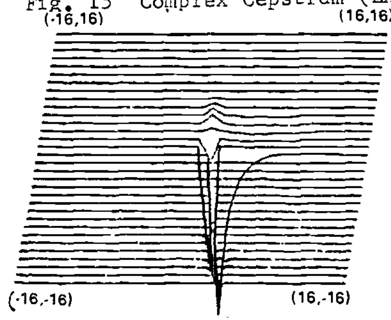

Fig. 17 Complex Cepstrum