

B-1  
#2

RECOGNITION OF OCCLUDED  
TWO DIMENSIONAL OBJECTS

Bir BHANU  
University of Southern  
California  
Los Angeles, USA

O.D. FAUGERAS  
INRIA  
Domaine de Voluceau  
B.P. 105  
78150 - LE CHESNAY

The problem of recognizing occluded or partially seen objects is becoming more and more important in applications such as biomedical image analysis, industrial inspection and Robotics. In this contribution we propose a hierarchical stochastic labeling technique to identify parts of two dimensional shapes represented by their polygonal approximations.

I) HIERARCHICAL STOCHASTIC LABELING :

We work with two polygons. One is the model  $M = \{M_1, \dots, M_N\}$  and one is the observed object  $O = \{O_1, \dots, O_{L-1}\}$  where  $M_i$  and  $O_j$  are line segments,  $i=1, \dots, N$  and  $j=1, \dots, L-1$ . We are trying to identify part of the model  $M$  within the observation  $O$ . We are therefore trying to label each of the segments  $M_i$  ( $i=1, \dots, N$ ) either as a segment  $O_j$  ( $j=1, \dots, L-1$ ) or as not belonging to  $O$  (label  $O_L = \text{NIL}$ ). Each segment  $M_i$  has therefore  $L$  possible labels.

Using a technique described in Section II we compute for every segment  $M_i$  a set of  $L$  positive numbers  $p_i(\ell)$ ,  $\ell=1, \dots, L$  forming a vector  $\vec{p}_i = [p_i(1), \dots, p_i(L)]^T$ .  $p_i(\ell)$  can be thought of as the probability of labeling the segment  $T_i, O_\ell$ . The set of vectors  $\vec{p}_i$  is called a stochastic labeling of the segments  $M_i$ .

Initially the stochastic labeling is ambiguous (except in some very special cases) and we make it evolve toward a less ambiguous labeling by comparing the local structures of  $M$  and  $O$ . From now on indexes  $i$  are taken modulo  $N$ . To every segment  $M_i$ , we associate the two neighboring segments  $M_{i-1}$  and  $M_{i+1}$ . In order to

compare the local structures of M and O we define two compatibility  $c_1(M_i, O_k, M_j, O_\ell)$  ( $j=i-1$  or  $i+1$ ) and  $c_2(M_i, O_k, M_{i-1}, O_\ell, M_{i+1}, O_m)$  which we denote more simply by  $c_1(i, k, j, \ell)$  and  $c_2(i, k, i-1, \ell, i+1, m)$ .

$c_1$  and  $c_2$  take values between 0 and 1.  $c_1(i, k, i-1, \ell)$  measures the ressemblance of the set  $\{M_i, M_{i-1}\}$  with the set  $\{O_k, O_\ell\}$ , for example. A good (bad) match means that the value of  $c_1$  is close to 1, (0).

As we describe in [1,2,3] we can associate to every segment  $M_i$  a compatibility vector  $\vec{q}_i = [q_i(1), \dots, q_i(L)]^T$ . Intuitively, this vector represents what the neighbors of segment  $M_i$  (that is to say segments  $M_{i-1}$  and  $M_{i+1}$ ) "think" about the way it should be labeled whereas  $\vec{p}_i$  represents what the segment  $M_i$  "thinks" about its own labeling.

Mathematically speaking, we compute

$$Q_{ij}(k) = \sum_{\ell=1}^L c_1(i, k, j, \ell) p_j(\ell) \quad \begin{array}{l} j=i-1, i+1 \\ i=1, \dots, N \\ k=1, \dots, L \end{array}$$

$$Q_i^{(1)}(k) = \frac{1}{2} (Q_{i \ i-1}(k) + Q_{i \ i+1}(k))$$

$$Q_i^{(2)}(k) = \sum_{\ell_1, \ell_2=1}^L c_2(i, k, i-1, \ell_1, i+1, \ell_2) p_{i-1}(\ell_1) p_{i+1}(\ell_2)$$

The numbers  $Q_i^{(1)}(k)$  and  $Q_i^{(2)}(k)$ ,  $k=1, \dots, L$  are positive. The idea is that they are large when the probabilities of the labels of the neighbors of  $M_i$  compatible with the label  $O_k$  are large and small otherwise. The numbers  $Q_i^{(1)}(k)$  and  $Q_i^{(2)}(k)$  are normalized so that they add up to 1 yielding two vectors  $\vec{q}_i^{(1)}$  and  $\vec{q}_i^{(2)}$  such that

$$q_i^{(j)}(k) = \frac{Q_i^{(j)}(k)}{\sum_{\ell=1}^L Q_i^{(j)}(\ell)} \quad \begin{array}{l} j=1, 2 \\ k=1, \dots, L \end{array}$$

The idea is to decrease the discrepancy between what every segment  $M_i$  thinks about its own labeling ( $\vec{p}_i$ ) and what its neighbors think about it ( $\vec{q}_i^{(j)}$ ,  $j=1, 2$ ). We have shown elsewhere [3,4] that a good

"local" measure of compatibility and nonambiguity is the inner product  $\vec{p}_i \cdot \vec{q}_i^{(j)}$  ( $j=1,2$ ). By computing the average over the set M of these local measure we obtain two global criteria :

$$J^{(j)} = \sum_{i=1}^N \vec{p}_i \cdot \vec{q}_i^{(j)} \quad j=1,2$$

The problem of labeling the segments  $M_i$  is therefore equivalent to an optimization problem : given an initial labeling  $\vec{p}_i^{(0)}$ ,  $i=1, \dots, N$ , find a local maximum of the criteria  $J^{(j)}$  ( $j=1,2$ ). Since  $c_2$  is a better measure than  $c_1$  of the local match between M and O we are actually interested in finding local maxima of the criterion  $J^{(2)}$ . On the other hand maximizing  $J^{(1)}$  is easier from the computational standpoint. We therefore use the following hierarchical approach : starting with an initial labeling  $\vec{p}_i^{(0)}$  we look for a local maximum  $\vec{p}_i^{(1)}$  of the criterion  $J^{(1)}$ . This labeling is less ambiguous than  $\vec{p}_i^{(0)}$  in the sense that many labels have been dropped (their probabilities  $p_i(k)$  are equal to zero). We then use the labeling  $\vec{p}_i^{(1)}$  as an initial labeling to find a local maximum of criterion  $J^{(2)}$ . The computational saving comes from the fact that the values  $c_2(i,k,i-1,l_1,i+1,l_2)$  corresponding to probabilities  $p_{i-1}(l_1)$  or  $p_{i+1}(l_2)$  equal to zero are not computed. Details about the maximization procedure can be found in [2,3,5,6]

## II) COMPUTING THE INITIAL PROBABILITIES AND THE COMPATIBILITY FUNCTIONS :

The initial probabilities are computed from a set of feature values such as length of a segment, angle between a segment and the horizontal axis, angle between two segments, etc. Let P be the number of features used. We measure the quality of the correspondance between the segments  $M_i$  and  $O_k$  as

$$C(M_i, O_k) = \sum_{p=1}^P |F_{mp} - F_{op}| W_p$$

where  $F_{mp}$  and  $F_{op}$  are the values of the p-th features of the model and the observation, respectively,  $W_p$  is a weight factor. The initial probabilities are then chosen proportional to  $\frac{1}{1+C(M_i, O_k)}$ .

The definition of the compatibility functions  $c_1$  and  $c_2$  is guided

by the type of deformation that we allow our polygons to undergo. We have restricted ourselves to rotation and scaling. For  $c_1$  for example, given two segments  $M_i$  and  $M_j$  in  $M$  and two segments  $O_k$  and  $O_\ell$  in  $O$  we compute the best transformation (composition of a rotation a change of scale and a translation) in the least squares sense that takes the pair  $(M_i, M_j)$  as close as possible to the pair  $(O_k, O_\ell)$ . If  $C_1(M_i, O_k, M_j, O_\ell)$  is the corresponding error we define

$$c_1(i, k, j, \ell) = \frac{1}{1 + C_1(M_i, O_k, M_j, O_\ell)}$$

$c_2$  is defined in a similar fashion. The problem of defining  $c_1$  and  $c_2$  when some of the segments in the observed object are equal to NIL is solved in [6].

### III) RESULTS, CONCLUSIONS :

In figure 1 we show the outline of a piece of a car shocks absorber. In figure 2 we show the outline of the superposition of two such pieces, the one below being the one of figure 1. From a practical standpoint, it is important to identify in the shape of figure 2 (the observation) the visible part of the shape of figure 1 (the model). Figures 3 and 4 show the corresponding polygonal approximations ( $N=L=28$ ). Table I shows the results of the hierarchical stochastic labeling algorithm at different iterations. We display only the label with the highest probability. The run time is 20 seconds on a DEC 10 machine.

In conclusion we have shown how the techniques of stochastic labeling could be successfully applied to the problem of recognizing partially visible 2D objects. We are in the process of extending our results to 3D.

### REFERENCES

- [1] O.D. Faugeras and M. Berthod, "Improving Consistency and Reducing Ambiguity in Stochastic Labeling : an Optimization Approach," to appear in IEEE Trans. Pattern Analysis and Machine Intelligence.
- [2] O.D. Faugeras, "Application des modèles de vision au traitement numérique des images," Thèse d'Etat, Université de Paris VI, 1979.
- [3] M. Berthod, "L'amélioration d'étiquetage : une approche pour l'utilisation du contexte en Reconnaissance des Formes," Thèse d'Etat, Université de Paris VI. 1980.
- [4] M. Berthod et O.D. Faugeras, "Using Context in the global Recognition of a set of objects : an optimization approach,"

- 8th World Computer Congress (IFIP 80) Tokyo, pp.695-698.
- [5] O.D. Faugeras, "Optimization Techniques in Image Analysis", Proceedings of the fourth International Conference on Analysis and Optimization of Systems, pp.790-823, Lecture Notes in Control and Information Sciences, Springer-Verlag, 1980.
- [6] B. Bhanu and O.D. Faugeras. "Shape matching using hierarchical gradient relaxation technique," USC-IPI Semi-Annual Technical #, pp.85-113.

Table 1. Labels of segments of M at different iterations of the maximization of criteria  $J^{(1)}$  and  $J^{(2)}$ .

Segments $M_i$	Labels at different iterations					
	0	4	8	4	8	12
1	1	1	1	1	1	1
2	28	28	28	28	28	28
3	28	28	28	28	2	2
4	28	28	28	28	28	28
5	28	28	28	28	28	28
6	28	28	28	28	28	28
7	28	28	28	28	28	28
8	28	28	25	25	25	25
9	28	26	26	26	26	26
10	27	27	27	27	27	27
11	28	1	1	28	28	28
12	8	8	8	8	8	8
13	28	28	9	9	9	9
14	28	28	28	28	28	11
15	28	28	28	28	28	28
16	28	28	28	28	28	28
17	28	28	28	28	28	28
18	28	28	28	28	28	28
19	28	28	28	28	28	28
20	28	28	28	28	28	28
21	28	28	28	28	28	28
22	28	28	28	28	28	28
23	28	22	22	22	22	22
24	23	23	23	23	23	23
25	24	24	24	24	24	24
26	25	25	25	25	25	25
27	26	26	26	26	26	26
28	27	27	27	27	27	27
Values of criteria	-	3.58	3.88	3.57	4.07	4.64

$J^{(1)}$

$J^{(2)}$

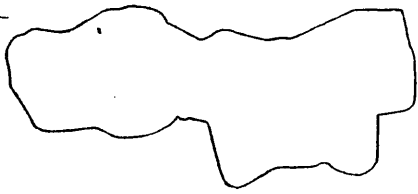


Figure 1.

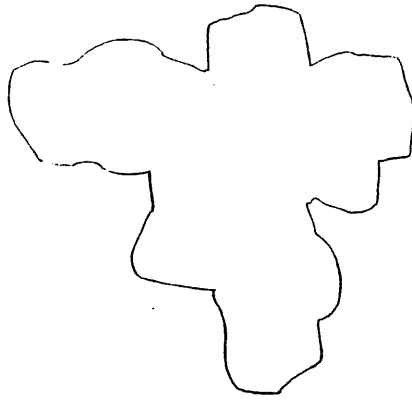


Figure 2.

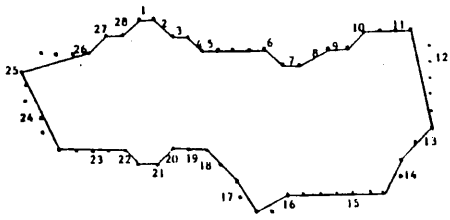


Figure 3.

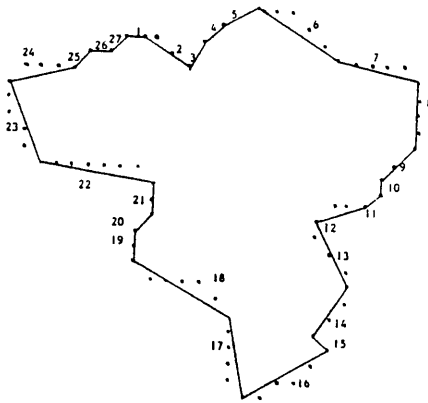


Figure 4.

Proceedings of  
**The Second  
Scandinavian  
Conference on  
Image Analysis**

Helsinki, Finland, June 15-17, 1981

Arranged by Suomen Hämmönnistyslaitoksen Seura -  
Pattern Recognition Society of Finland



*E. Oja*

# Proceedings of The Second Scandinavian Conference on Image Analysis

Helsinki , Finland 15 - 17.6.1981

Arranged by Suomen Hämmöntunnistustutkimuksen seura ry.  
Pattern Recognition Society of Finland

Sponsored by International Association for Pattern  
Recognition ( IAPR )

Edited by:  
Erkki Oja  
Olli Simula